

MA 8109 SUMMARY LEC 2

Int. theory $\rightarrow$ probab. theory $\rightarrow$ stoch. proc. $\rightarrow$ stoch. int. $\rightarrow$ SDEs $\rightarrow \dots$

Measure space $(X, \mathcal{F}, m)$
set $\uparrow$ σ -alg. measure

$f : X \rightarrow Y$ $\mathcal{F}$ -measurable if $f^{-1}(A) \in \mathcal{F}$ for all $A \in \mathcal{B}_Y$
Borel σ -alg.

Sum, product, max, sup, lim, limsup of meas. func'ns are meas.

Simple function $s(x) = \sum_{i=1}^n a_i \chi_{A_i}(x)$, $a_i \in \mathbb{R}$, $A_i \in \mathcal{F}$

Lebesgue integral of $f \geq 0$: $\int f \, dm = \sup \left\{ \int s \, dm : s \text{ simple, } 0 \leq s \leq f \right\}$
" $\sum a_i m(A_i)$

Interchange lim and $\int$: $\int \lim_{i \rightarrow \infty} f_i \, dm = \lim_{i \rightarrow \infty} \int f_i \, dm$ if

All f_i 's $\mathcal{F}$ -meas.!

MCT: $0 \leq f_1 \leq f_2 \leq \dots \leq f_i \xrightarrow{\text{a.e.}} f$
monotone conv. thm.

OR LDCT: $|f_i| \leq h$, $\int h \, dm < \infty$, $f_i \xrightarrow{\text{a.e.}} f$
dominated conv. thm.